VOTING MANIPULATION

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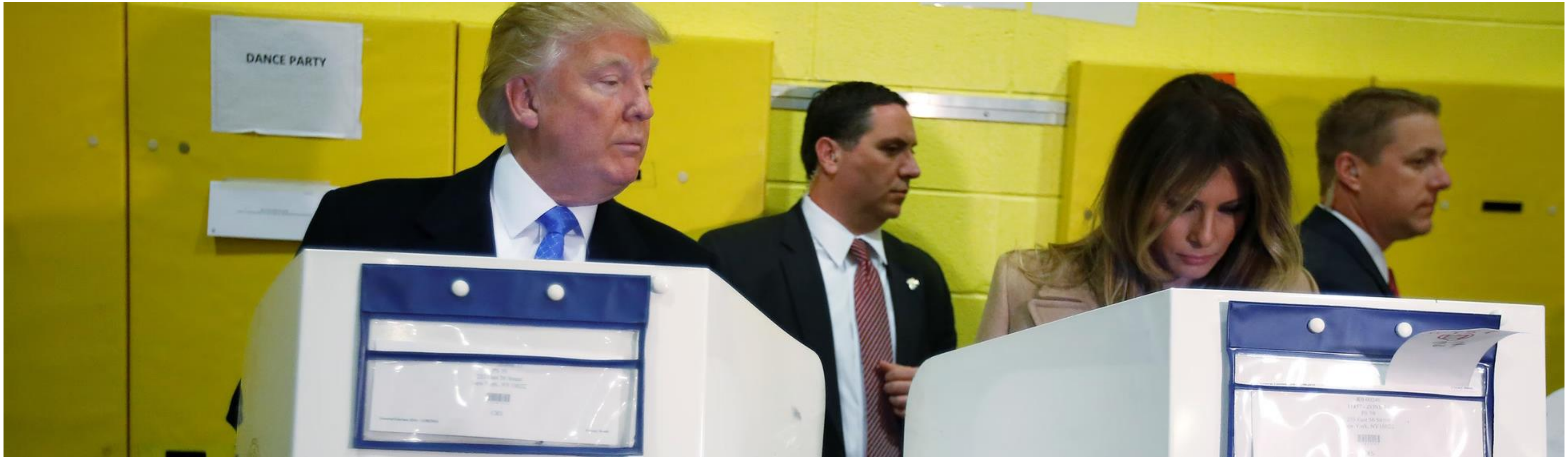
AGENDA

- Motivation
- Voting and Manipulation –
  - Gibbard–Satterthwaite Theorem
  - Strategic voting
- Computational complexity of existing voting rules
  - Qualitative comparison of manipulability



SOCIAL CHOICE AND VOTING MANIPULATION

- All voting rules that are not dictatorial and consist of more than two outcomes are manipulable.
- There are many ways in which voters can manipulate their votes to secure a more favorable outcome.
- If all voting rules can be manipulated in some way, how can we still ensure a fair voting system?

THE PROBLEM

GIBBARD–SATTERTHWAITE THEOREM

- For any social choice function, if there are more than two possible outcomes and any strict ranking of these alternatives is permissible. Then the only unanimous, strategy-proof social choice function is a dictatorship. Proof

STRATEGIC VOTING

COMPROMISING

- When a voter ranks an alternative higher in the hope of getting that candidate elected.

voter	1^{st}	2^{nd}	3^{rd}	4^{th}
Alice	A	B	C	D
Bob	C	B	D	A
Charlie	C	B	D	A

Vote count : {A: 3, B: 6, C: 7, D: 2 }

voter	1^{st}	2^{nd}	3^{rd}	4^{th}
Alice	B	A	D	C
Bob	C	B	D	A
Charlie	C	B	D	A

Vote count : {A: 2, B: 7, C: 6, D: 2 }

BURYING

- A voter insincerely ranks an alternative lower in the hopes of defeating it.

voter	1st	2nd	3rd	4th
Alice	D	B	C	A
Bob	B	C	A	D
Charlie	C	A	D	B

→

voter	1st	2nd	3rd	4th
Alice	D	B	A	C
Bob	B	C	A	D
Charlie	C	A	D	B

Vote count : {A: 3, B: 5, C: 5, D: 4 }

Vote count : {A: 4, B: 5, C: 4, D: 4 }

PUSH OVER

- A voter ranks a perceived weak alternative above their preferred candidate, in order to actually elect the preferred candidate and not the weak candidate.

voter	1^{st}	2^{nd}	3^{rd}	4^{th}
Alice	C	B	A	D
Bob	B	C	D	A
Charlie	C	B	A	D
David	B	C	A	D

Vote count : {A:3, B: 10, C: 10, D: 1 }

voter	1^{st}	2^{nd}	3^{rd}	4^{th}
Alice	D	C	A	B
Bob	B	C	D	A
Charlie	C	B	A	D
David	B	C	A	D

Vote count : {A:3, B: 8, C: 9, D: 4 }

BULLET VOTING

- A voter selects just one candidate, despite having the option to vote for more than one.

voter	1st	2nd	3rd	4th
Alice	D	B	C	A
Bob	B	A	D	C
Charlie	C	A	D	B

voter	1st	2nd	3rd	4th
Alice	D			
Bob	B	A	D	C
Charlie	C	A	D	B

Vote count : {A:4, B: 5, C: 4, D: 5 }

Vote count : {A:4, B: 3, C: 4, D: 5 }

REAL LIFE EXAMPLES

and even more examples can be found here

- **Canada** – In the 2004 federal election, and in the 2006 election, strategic voting was a concern for the federal New Democratic Party (NDP). In the 2004 election, the governing Liberal Party was able to convince many New Democratic voters to vote Liberal to avoid a Conservative government.
- **France** – The two-round system in France shows strategic voting in the first round, due to considerations which candidate will reach the second round.
- **UK** – In the 2017 general election, it is estimated that more than 20% of voters voted tactically either as a way of preventing a "hard Brexit" or preventing another Conservative government. Many Green Party candidates withdrew from the race in order to help the Labour Party secure closely fought seats against the Conservatives.
- **Hong Kong** – In Hong Kong, with its party-list proportional representation using largest remainder method with the Hare quota, voters supporting candidates of the pro-democracy camp often organize to divide their votes across different tickets, avoiding the concentration of votes on one or a few candidates.

NP-COMPLETE VOTING SCHEMES

THE IDEA

- In this setting the agents are computers agents, as opposed to humans due to faster and more efficient computation. They are also inherently more objective and are not affected by irrational motives.
- By developing NP-complete voting schemes manipulators will need to perform NP-hard computations to manipulate the system, thus decreasing their likelihood.

THE SETTING

- **Complete information** – Assuming Complete information can then be extended to the incomplete case, as any hardness results will be applied to the more complex setting of incomplete information.
- **Coalition Manipulation** – In large election systems it is unlikely that a single voter can manipulate the results on their own.
- **Weighted Votes** – Manipulation by a weighted coalition can be used to prove the hardness of the unweighted (but correlated) voters in the incomplete case.

NP-COMPLETE VOTING SCHEMES

- In order to prove NP-hard results they used a reduction from the PARTITION problem to a constructive or destructive manipulation.
- **Constructive manipulation** – a voter is trying make a candidate win the election.
- **Destructive manipulation** – a voter is trying to prevent a candidate from winning the election.

RESULTS

Constructive Manipulation

# of Candidates	2	3	4,5,6	≥ 7
Borda	P	NP-c	NP-c	NP-c
Veto	P	NP-c	NP-c	NP-c
STV	P	NP-c	NP-c	NP-c
Plurality with runoff	P	NP-c	NP-c	NP-c
Copeland	P	P	NP-c	NP-c
Maxmin	P	P	NP-c	NP-c
Randomized cup	P	P	P	NP-c
Regular cup	P	P	P	P
plurality	P	P	P	P

Destructive Manipulation

# of Candidates	2	≥ 3
STV	P	NP-c
Plurality with runoff	P	NP-c
Randomized Cup	P	?
Borda	P	P
Veto	P	P
Copeland	P	P
maxmin	P	P
Regular cup	P	P
plurality	P	P

ESTIMATING THE DEGREE OF MANIPULABILITY

- Given a set of m outcomes and n agents, there are $m!$ possible ordering of preference profiles and $(m!)^n$ total possible profile ordering for n agents.
- Manipulability index (Kelly's index)
- $K = \frac{d_0}{(m!)^n}$
- Where d_0 is the number of profiles where manipulation can occur.

RESULTS

- The manipulability index was calculated for 5 rules, with $m = 5$

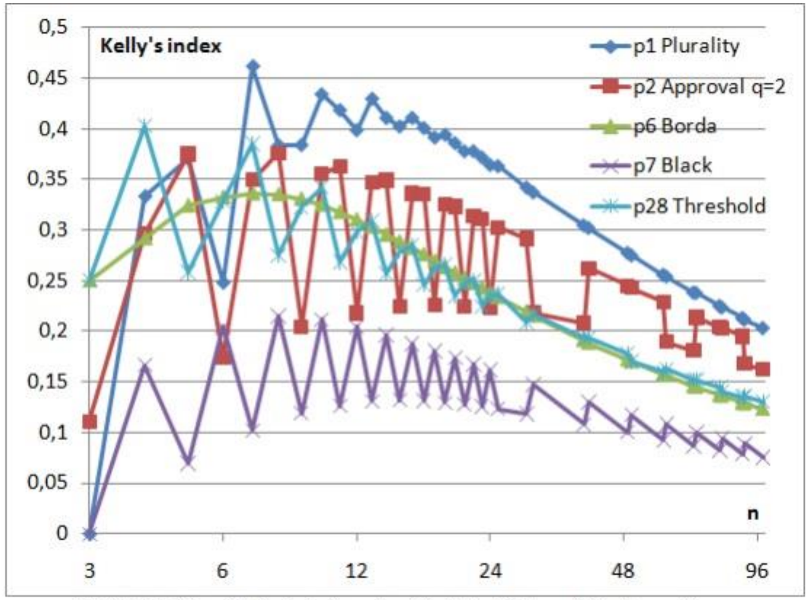


FIGURE 1. Kelly's index for Kelly's DA and 3 alternatives

The results are very similar to [2]. Black procedure is the least manipulable rule for almost all number of voters and for rules such as Plurality, Approval voting and Threshold, there is a cycle length of m.

ANOTHER WAY TO COMPARE

- The circled profiles have an incentive to submit a false ballot

	x y z	x z y	y x z	y z x	z x y	z y x
xyz	x	x	x	y	x	x
xzy	x	x	x	x	x	z
yxz	x	x	y	y	x	y
yzx	y	x	y	y	z	y
zxy	x	x	x	z	z	z
zyx	x	z	y	y	z	z

Fig. 1

ANOTHER WAY TO COMPARE

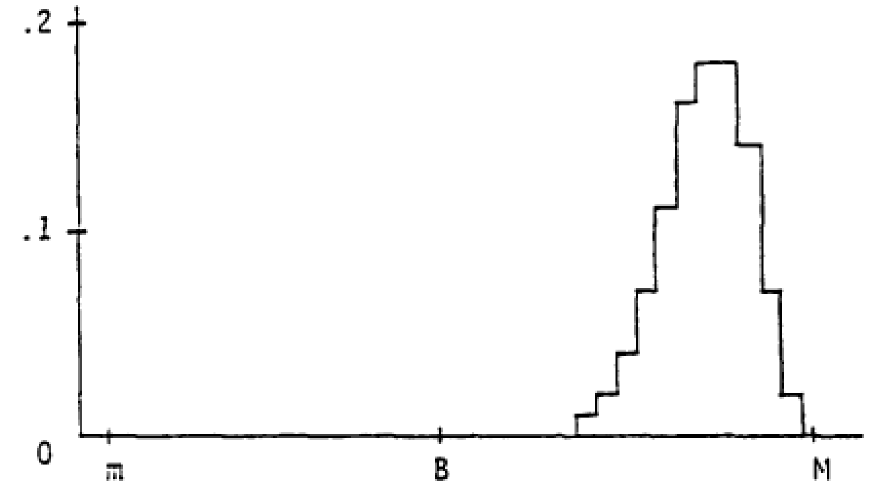
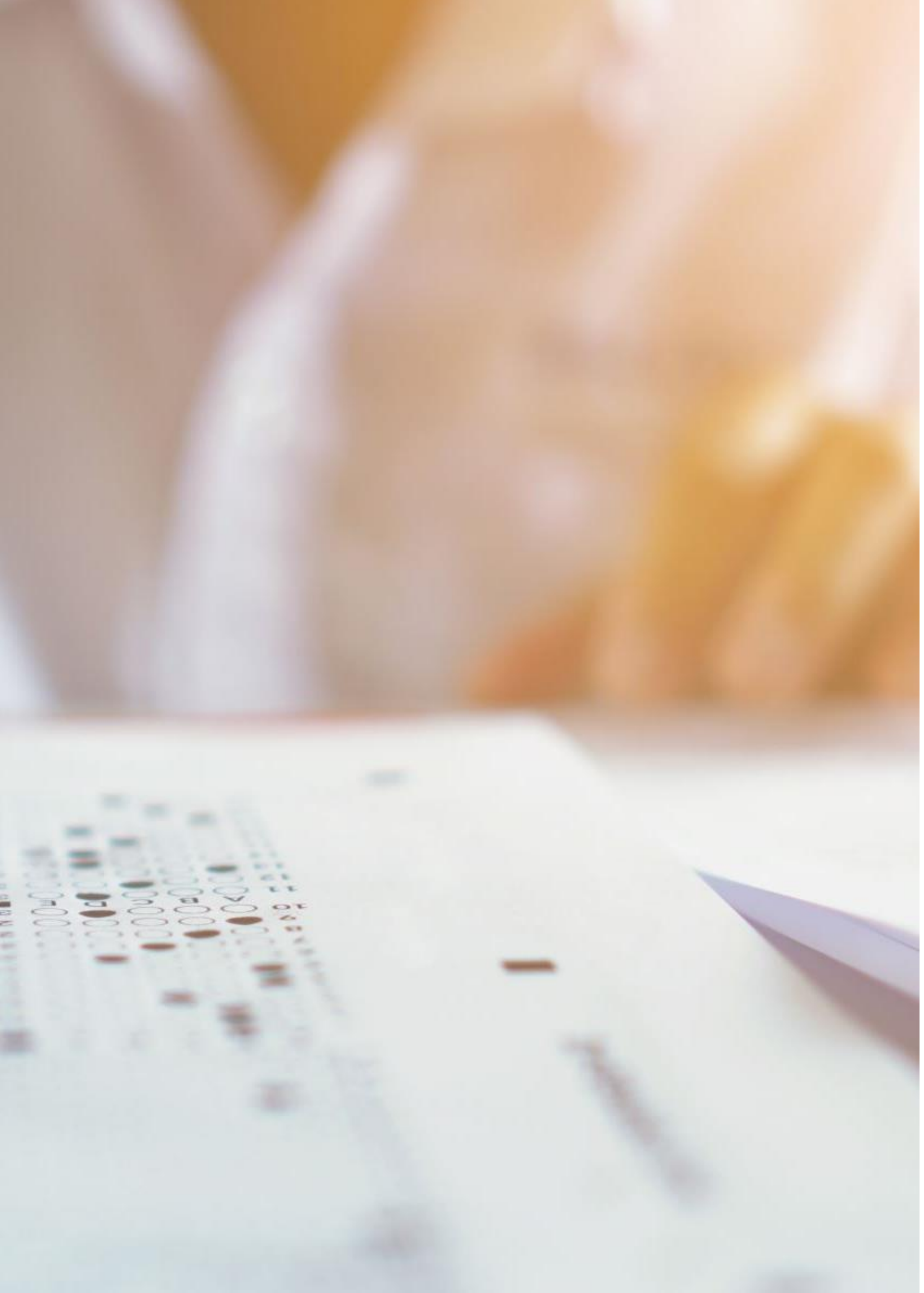


Fig. 2. Degrees of manipulability: Two agents, three alternatives, strong orders

B – Manipulability degree of Borda,

M - Maxmin

QUESTIONS & DISCUSSION



TAKEAWAYS

- No voting rule is perfect, manipulability is unavoidable
- We can however make manipulation extremely hard or unlikely
- When choosing a voting rule we should take into account the demographic and consider the trade-off between possible outcomes and likely outcomes.

REFERENCES

- Cross E. NP-Hard Manipulations of Voting Schemes.
- Zuckerman M, Lev O, Rosenschein JS. An algorithm for the coalitional manipulation problem under maximin. InThe 10th International Conference on Autonomous Agents and Multiagent Systems-Volume 2 2011 May 2 (pp. 845-852).
- Taylor AD. Social Choice and the Mathematics of Manipulation. Cambridge: Cambridge University Press; 2005. (Outlooks).
- Lepelley D, Mbih B. The vulnerability of four social choice functions to coalitional manipulation of preferences. Social Choice and Welfare. 1994 Jul;11(3):253-65.
- Karabekyan D, Yakuba V. Estimating the degree of manipulability of voting rules for weak manipulation. InAIP Conference Proceedings 2010 Sep 30 (Vol. 1281, No. 1, pp. 2151-2154). American Institute of Physics.